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VIEWING ANGLE AND ENVIRONMENT EFFECTS IN GRB: SOURCES OF AFTERGLOW DIVERSITY

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ABSTRACT

We discuss the afterglows from the evolution of both spherical and anisotropic fireballs decelerating in an inhomogeneous external medium. We consider both the radiative and adiabatic evolution regimes, and analyze the physical conditions under which these regimes can be used. Afterglows may be expected to differ widely among themselves, depending on the angular anisotropy of the fireball and the properties of the environment. They may be entirely absent, or may be detected without a corresponding γ-ray event. A tabulation of different representative light curves is presented, covering a wide range of behaviors that resemble what is currently observed in GRB 970228, GRB 970508 and other objects.

Subject headings: gamma-rays: bursts

1. Introduction

The discovery of the afterglows of gamma-ray bursts (GRB) provides new information which can constrain the models used to explain these objects. Significant interest was aroused by the fact that several of the features reported in the first GRB detected over time scales $\gtrsim$ days at X-ray (X) and optical (O) wavelengths, GRB 970228 (Costa et al, 1997a) agreed quite well with theoretical expectations from the simplest relativistic fireball afterglow models published in advance of the observations (Mészáros & Rees, 1997a; see also Vietri, 1997a). A number of theoretical papers were stimulated by this and subsequent observations (e.g. Tavani, 1997; Waxman, 1997a;Reichart, 1997; Wijers, Rees & Mészáros , 1997, among others), and interest continued to grow as new observations provided controversial evidence for the distance scale and the possible host (Sahu et al, 1997). New evidence and new puzzles were added when the optical counterpart to the second discovered afterglow (GRB 970508) was attributed a cosmological redshift (Metzger et al., 1997), as well as a radio counterpart (Frail, et al, 1997;Taylor, et al, 1997) and X/O light curves showing

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a rise and decline (Djorgovski, S. et al., 1997;Fruchter, et al, 1997). Some bursts, however, were detected only in X but not O (e.g. GRB 970828), while some which would have been expected to be seen in X or O were not (e.g. GRB 970111).

Additional structure on the light curves has also emerged from a continued analysis of some of these objects down to the faintest flux levels. The large variety of behaviors exhibited by afterglows, while clearly compatible with relativistic fireball models, poses new challenges of interpretation, e.g. Waxman, 1997b;Vietri, 1997b;Katz & Piran, 1997;Rhoads, 1997; Paczyński, 1997. Some of the questions at the forefront of attention include the effect of the external medium, the degree to which afterglows may be considered to be isotropic events, and the effects of the radiative efficiency on the evolution of the remnant. We address all three of these issues here. We also clarify some of the issues that have been recently raised about the dynamical effects of different radiative efficiency regimes. We then discuss the possible variety of afterglow behavior that is expected from isotropic or anisotropic fireballs expanding in a medium which may be inhomogeneous, either due to external gradients or due to expansion in an irregular cavity. We apply these models to interpret some of the salient observational features of several GRB afterglows, and discuss their possible use for predicting detection rates of X/O/R afterglows undetected in γ-rays, as well as some possible reasons for the non-detection of afterglows in GRB.

2. Expansion Dynamics and Radiative Efficiency

In some bursts (e.g. GRB 970508) the afterglow seems to contain a significant amount of energy compared to the typical (isotropic) estimate of $E \sim 10^{51}$ erg s^{-1}. This led Vietri, 1997b to suggest that the afterglow must remain radiatively efficient $\sim$ weeks after the burst and evolve with $\Gamma \propto r^{-3}$. This regime was also considered in Katz & Piran, 1997, who refer to previous relativistic fireball models as radiating only a small fraction of the total kinetic energy, and go on to consider instantaneously cooling fireballs. It is important to discuss in more detail what are the conditions necessary for the radiative efficiency having an effect on the dynamic evolution of an expanding cloud.

The classical fireball models, with "isotropic equivalent" energies of $E \sim 10^{51}$ erg and bulk Lorentz factors $\Gamma \sim 10^2 - 10^3$ have, in fact, been generally been taken to be in the radiative stage during the γ-ray event, i.e. radiative efficiency near unity, meaning that of order the initial total kinetic energy of the protons is radiated in the observer-frame expansion time (Rees & Mészáros , 1992, and subsequent papers). However, for some parameters the bulk of this energy can appear at energies other than MeV (Mészáros , Rees & Papathanassiou, 1994;Mészáros & Rees, 1994). This high efficiency in the initial deceleration shock can occur if the electrons are assumed *in the shock* to be heated to $\gamma_e \sim \xi_e (m_p/m_e)\Gamma$ with ξ_e reasonably close to unity. Experimental evidence from interplanetary collisionless shocks indicates that this could be the case. In such fireballs the electrons are likely to retain high radiative efficiency for some time after the GRB, and energetically the most important are the newly shocked electrons near the downwards evolving

peak, of initial post-shock energy $\gamma_e \sim \xi_e (m_p/m_e)\Gamma(t)$. This peak is generally where most of the electron internal energy is. In the regime where the peak electrons have high radiative efficiency, *if throughout the entire remnant volume the protons are able to establish (and remain in) equipartition with the electrons*, then the remnant evolves with $\Gamma \propto r^{-3}$ in a homogeneous external medium (e.g. Blandford & McKee, 1976; Vietri, 1997a; Katz & Piran, 1997). This follows simply from the momentum conservation law, if we can assume that the radiative losses tap also the proton and magnetic energy (strong coupling), and the radiative time scale is shorter than the dynamic time scale, so that energy conservation cannot be used. This is the classical "snowplow" approximation of supernova remnants. The alternative regime is that where the radiative losses do not tap the proton and magnetic energy, only the electron energy (weak coupling), and/or the radiative cooling time scale is longer than the dynamic time scale. In this case, possibly after an initial short cooling of the electrons, one can assume energy conservation (most of the energy is in the protons and/or magnetic fields), and one has $\Gamma \propto r^{-3/2}$ in a homogeneous medium (e.g. Blandford & McKee, 1976, Paczyński & Rhoads, 1993, Katz, 1994b, Mészáros & Rees, 1997a).

The (strong coupling) radiative regime and the adiabatic regime can be generalized to the case where the fireball moves into an inhomogeneous external medium. We consider a spherical fireball of energy E and bulk Lorentz factor Γ which are independent of angle θ, expanding into an external medium of density $n(r) \propto r^{-d}$ which is also independent of the angle, where r is distance from the center of the explosion. The shocked gas evolves according to the conservation law

$$\Gamma^{1+A} r^3 n \propto \Gamma^{1+A} r^{3-d} \propto \text{constant} , \tag{1}$$

where $A = 1(0)$ corresponds to energy(momentum) conservation, i.e. to the adiabatic (radiative) regimes. These regimes must be understood in a global or dynamic sense, as applying to the entire remnant, i.e. baryons, magnetic fields, electrons, etc., or at any rate to its dynamically dominant constituents. Since the observer-frame or detector time t must satisfy $r \propto ct\Gamma^2$, we have

$$\Gamma \propto r^{-(3-d)/(1+A)} \propto t^{-(3-d)/(7+A-2d)} \quad , \quad r \propto t^{(1+A)/(7+A-2d)} . \tag{2}$$

If $d = 0$ and a remnant starts out in the strong coupling radiative regime ($A = 0$) then $\Gamma \propto r^{-3}$. However after the expansion has proceeded for some time eventually the cooling time of the electrons at the peak of the distribution becomes longer than the expansion time, at which point energy conservation (adiabatic approximation) becomes valid (eq.1 with $A = 1$) leading to $\Gamma \propto r^{-3/2}$. The power law of equation (2) is valid as long as the remnant is relativistic, i.e., until the time $t_{nr}/t_o \sim \Gamma_o^{(7+A-2d)/(3-d)}$, where $t_o \sim t_\gamma$ is the duration of the γ-ray burst itself and Γ_o is the initial Lorentz during the burst. For a homogeneous medium $d = 0$ and $t_{nr}/t_o \sim \Gamma_o^{(7+A)/3}$, e.g. for an adiabatic remnant $A = 1$ with $\Gamma_o = 10^3$ and $t_\gamma = 1$ s, one has $t_{nr} \sim 1$ year. The subsequent behavior in the nonrelativistic stage is described in Wijers, Rees & Mészáros , 1997. Shorter times for reaching the nonrelativistic stage are possible for a radiative remnant, or for a radiative stage followed by an adiabatic one, while longer times can occur for longer t_γ or for expansion into a medium whose density decreases with distance ($0 \leq d \leq 3$).

However, the conditions under which the remnant dynamics may be considered adiabatic or radiative is far from unambiguous, and is crucially dependent on poorly known questions about post-shock energy exchange between protons and electrons. Electrons cool quickly compared to protons, and it remains an unsolved question, of importance also in other areas of astrophysics, whether *behind the shocks*, after the electrons have cooled, the protons remain hot (i.e. a two temperature plasma, as in hot torus models of AGN), or whether they tend toward some degree of equipartition with the cooled electrons by virtue of unknown fast energy exchange mechanisms. The collisionless shock transition is the only place where one is guaranteed fast varying chaotic electric and magnetic fields which can lead to quick relaxation, and it is known that an interplay between protons and magnetic fields can occur there which can lead to values close to equipartition. Such equipartition is also inferred from measurements in the ISM. However, in fast evolving flows such as those in GRB, it is unclear whether such equipartition can occur anywhere except possibly near the shock transition itself. If behind the shocks the protons are unable to quickly readjust to the electron losses, then even if the electrons are radiative ($a = 1$) the protons can be adiabatic ($A = 1$) leading to $\Gamma \propto r^{-3/2}$, in a homogeneous external medium. A related problem is that, in order for the remnant to evolve with $\Gamma \propto r^{-3}$ (again for a homogeneous external medium), the magnetic energy should also decrease, since if the latter were conserved it would soon dominate the total energy density and the remnant would evolve as a polytrope with adiabatic index 4/3 which leads to $\Gamma \propto r^{-3/2}$. Thus both the proton and the magnetic energy need to be transferred on a fast time scale to the electrons in order to ensure $\Gamma \propto r^{-3}$. An additional factor that might contribute towards a steepening of the decay of Γ are other energy losses, e.g. such as from the escape of accelerated nonthermal protons from the shell, if these carry substantial energy. So far, there are neither detailed simulations nor experimental evidence concerning this in GRB. In the absence of such losses, or of a quick energy exchange between post-shock protons plus magnetic fields and the electrons, one can therefore have a situation where the electrons are "radiatively efficient", but the dynamics of the remnant expansion follows an "adiabatic" law. This occurs if the electron cooling time is less than the expansion time. The shocked electrons can radiate up to half of the proton energy in the shocks, but the protons and the magnetic fields retain at least half and this would be enough to ensure a quasi-adiabatic dynamic evolution of the remnant with $\Gamma \propto r^{-3/2}$. The latter is also true when the electrons are adiabatic. For an inhomogeneous medium that decays with radius, both of these decay laws would be flatter.

There is no difficulty in treating the weak coupling case where the electrons are radiatively efficient but the dynamics is adiabatic (§3). This regime is physically as plausible, if not more, as the strong coupling one where protons and fields exchange energy with electrons on a fast time scale. For reasons of simplicity, in §§3, 4 and in the rest of the paper we will assume that magnetic fields are near equipartition with the protons, which ensures a simple expression for the electron radiative efficiency in terms of only the synchrotron cooling time and the expansion time (the situation where inverse Compton (IC) cooling is important only introduces some extra changes in the way the synchrotron efficiency is defined).

3. Spherical Inhomogeneous Models

As in the previous section, we consider a spherical fireball of energy E and bulk Lorentz factor Γ independent of angle θ expanding into an external medium of density $n(r) \propto r^{-d}$. In the simplest afterglow model one considers the time evolution of the radiation from the external medium shocked by the blast wave as it slows down. Denoting quantities in the comoving frame of the shocked fluid with primes, in the post-shock region the density is $n' \sim n\Gamma$, the mean proton and electron random Lorentz factors are $\gamma_p \sim \Gamma$ and $\gamma_e \sim \xi_e(m_p/m_e)\Gamma$ (where $(m_e/m_p) \lesssim \xi_e \lesssim 1$ is the fraction of the electron equipartition energy relative to protons), the turbulently generated magnetic field (assumed to build up to a fraction $\xi_B \leq 1$ of the field in equipartition with the random proton energy) is $B' \propto \xi_B n^{1/2}\Gamma$, and the peak of the electron synchrotron spectrum is at comoving frequency $\nu'_m \propto B'\gamma_e^2 \propto \xi_B\xi_e^2 n^{1/2}\Gamma^3 \propto r^{-d/2}\Gamma^3$. The corresponding observer peak frequency is

$$\nu_m \propto \xi_B\xi_e^2 n^{1/2}\Gamma^4 \propto t^{-[12-(d/2)(7-A)]/(7+A-2d)} \tag{3}$$

The synchrotron radiative efficiency at ν_m is $e_{sy,m} \sim t'^{-1}_{sy,m}/(t'^{-1}_{sy,m} + t'^{-1}_{ex} + t'^{-1}_{other})$ where $t'_{sy,m} \propto 1/(\xi_e\xi_B^2\Gamma^3 n)$ is the comoving synchrotron cooling time at ν_m, t'_{ex} is comoving expansion time or adiabatic cooling time and t'_{other} is any other loss mechanism, e.g. inverse Compton (IC), if important. In the limit where only synchrotron and/or adiabatic losses are important we may write $e_{sy,m} \sim (t'_{ex}/t'_{sy,m})^a$, which is unity in the electron radiative ($a = 0$) regime, and ≤ 1 in the electron adiabatic ($a = 1$) regime. We have $e_{sy,m} \sim (\xi_B^2\xi_e r n\Gamma^2)^a \propto (\xi_B^2\xi_e r^{1-d}\Gamma^2)^a$.

We first assume (§2) that protons and magnetic fields are strongly coupled to electrons, so if the electrons are radiative the entire remnant is radiative, and the index $A = a$. The comoving synchrotron intensity at the comoving peak frequency is $I'_{\nu'_m} \propto n'_e(P'_{sy}/\nu'_m)ct'_{min} \sim n'_e(\gamma_e m_e c^2/\nu'_m)ce_{sy,m} \propto \xi_B^{2a-1}\xi_e^{a-1}r^{a-d(a+1/2)}\Gamma^{2a-1}$, where $t'_{min} \sim t'_{sy}e_{sy,m}$ is the shortest of the possible cooling times (synchrotron or adiabatic, in the above approximation). The flux from the relativistically expanding source at observer frequency ν_m is $F_{\nu_m} \propto t^2\Gamma^5 I'_{\nu_m}$, or

$$F_{\nu_m} \propto \nu_m^{-[2(1-a)-(d/2)(1+a)]/[12-(d/2)(7-a)]} \;\propto\; t^{[2(1-a)-(d/2)(1+a)]/(7+a-2d)} \;, \tag{4}$$

scaling with $\xi_e^{a-1}\xi_B^{2a-1}$. If the expansion is in the radiative $a = 0$ regime, $F_{\nu_m} \propto \nu_m^{-[2-(d/2)]/[12-(7/2)d]} \propto t^{(4-d)/(14-4d)}$ increases in time for any $d < 3$, being $\propto \nu_m^{-1/6} \propto t^{2/7}$ in a homogeneous medium with $d = 0$ (Vietri 1997b obtains a different scaling by taking in I'_{ν_m} the shocked gas comoving width $\Delta R'$ as path length, which however is equal to ct'_{min} only for adiabatic expansion, e.g. Mészáros & Rees, 1997a, where $t'_{min} \sim t'_{ex}$). In the adiabatic $a = 1$ regime $F_{\nu_m} \propto \nu_m^{d/(12-3d)} \propto t^{-d/(8-2d)}$, which is a constant independent of ν_m and of time for adiabatic expansion in a homogeneous $d = 0$ medium (Mészáros & Rees, 1997a;Katz, 1994b), but F_{ν_m} decreases in time for an inhomogeneous medium with $0 < d < 3$ (for $d \geq 3$ the fireball encounters most of the external mass near its initial radius). For a power-law spectrum $F_\nu \propto \nu^\alpha$, the flux at a fixed detector frequency ν_D is

$$F_D = F_{\nu_m}(\nu_D/\nu_m)^\alpha \;\propto\; t^{[2(1-a)+12\alpha-(d/2)(\{1+a\}+\alpha\{7-a\})]/[7+a-2d]} \;. \tag{5}$$

scaling as $\xi_e^{-1+a-2\alpha}\xi_B^{-(1+\alpha)+2a}$. Eq. (5) is valid for the strong electron-proton coupling regime. For a typical synchrotron spectrum $\alpha \simeq 1/3(-1)$ below(above) the break frequency $\nu_m(t)$, as the latter decreases in time the flux from radiative $a = 0$ models in the detector frequency band ν_D at times for which $\nu_D < \nu_m$,$\nu_D > \nu_m$ is $F_D \propto t^{6/7},\ t^{-10/7}$ in a homogeneous $d = 0$ medium, and $F_D \propto t^{8/9},\ t^{4/3}$ in an inhomogeneous $d = 2$ medium. Adiabatic $a = 1$ models give $F_D \propto t^{1/2},\ t^{-3/2}$ in a homogeneous medium, and $F_D \propto t^0,\ t^{-2}$ in an inhomogeneous $d = 2$ medium. Other values can be calculated from eq. (5) for different α before and after the break. Tables 1 and 2 give for the isotropic strong coupling case several examples in the next to last column.

A different regime is obtained if one assumes that the protons and magnetic fields are *not* strongly coupled to the electrons behind the shocks. In this case the dynamics of the remnant as a whole is controlled by the index A in eqs. (1,2), and as long as $\Gamma \gg 1$ the remnant is adiabatic with $A = 1$ and $\Gamma \propto r^{-3/2}$ for a homogeneous medium, whether the electrons are radiative or not, i.e. independent of a (see §2). More generally, in this case $\Gamma \propto t^{-(3-d)/(8-2d)}$, $r \propto t^{2/(8-2d)}$), and

$$\nu_m \propto \xi_B \xi_e^2\ t^{-(12-3d)/(8-2d)}\ . \tag{6}$$

In the synchrotron efficiency one must keep a separate index $a \neq A$ to account for the electrons being radiative or not. We have then $I'_{\nu_m} \propto \xi_B^{2a-1}\xi_e^{a-1}r^{a(1-d)-d/2}\Gamma^{2a-1}$, and $F_{\nu_m} \sim t^2\Gamma^5 I'_{\nu_m}$ is

$$F_{\nu_m} \propto \nu_m^{-[4(1-a)-d]/(12-3d)}\ \propto\ \xi_B^{2a-1}\xi_e^{a-1}\ t^{[4(1-a)-d]/(8-2d)}\ . \tag{7}$$

In a given fixed detector band ν_D one observes for a typical spectral shape $F_\nu \propto \nu^\alpha$ a time-dependent flux $F_D \propto F_{\nu_m}\nu_m^{-\alpha}$, or

$$F_D \propto \xi_B^{(2a-1-\alpha)}\xi_e^{a-1-2\alpha}\ t^{[4(1-a)-d+\alpha(12-3d)]/(8-2d)} \tag{8}$$

This is valid in the relativistic expansion regime. After a remnant becomes nonrelativistic (see below equation 2), the flux in a homogeneous $d = 0,\ a = 1$ medium would steepen to $F_D \propto t^{(3+15\alpha)/5} \propto t^{-12/5}$ for $\alpha = -1$ (Wijers, Rees & Mészáros , 1997). The variety of time behaviors possible for relativistic isotropic models in both the strong and weak coupling cases depending on whether they are radiative or adiabatic is shown in the last two columns of Tables 1 and 2.

The time behavior given by eqs. (5,8) can be complicated by at least two effects. One is that, unless observations start after the peak electrons are adiabatic, at some subsequent time the value of a in eqs. (4,5,7,8) switches from 0 to 1 as the peak electrons become adiabatic. The second is that if the detector frequency $\nu_D > \nu_m$,the flow (being controlled by particles radiating at ν_m) can already be adiabatic, while the smaller number of higher energy particles radiating at ν_D may still radiate efficiently. The frequency and time power law dependences of the flux before and after electron radiative inefficiency occurs are different. For a steady injection of accelerated electrons, the self-consistent electron energy power law index p in the presence of fast synchrotron losses is one power steeper than the injected spectrum, and the self-consistent synchrotron power law index $\alpha = (p-1)/2$ is a half power steeper than for the adiabatic (negligible loss) case. If

the lowest energy electrons near the peak $\gamma_e \sim \xi_e(m_p/m_e)\Gamma$ are radiatively efficient, all electrons above that are as well. As the remnant evolves, the first electrons to become inefficient are the lowest energy ones (in the peak corresponding to $\nu_m(t)$), and a flattening break by 1/2 power in the photon spectrum at frequency $\nu_b(t) > \nu_m(t)$ moves to frequencies increasingly higher than $\nu_m(t)$. The electron Lorentz factor at which the synchrotron time just equals the expansion time $r/c\Gamma$ is $\gamma_b \propto r^{[2-A(1-d)]/(1+A)}$ and the corresponding "adiabatic" photon frequency is

$$\nu_b \propto B'\gamma_b^2\Gamma \propto n^{1/2}\Gamma^2\gamma_b^2 \propto r^{-[2-(3/2)d]} \propto t^{-[2-(3/2)d](1+A)/(7+A-d)} , \tag{9}$$

which can either decrease in time for $d < 4/3$ (including a homogeneous medium with $d = 0$) or increase for $d > 4/3$ (although it always increases with respect to $\nu_m(t)$). For an external medium whose density drops with radius faster than $d \gtrsim 4/3$, if initially $\nu_b > \nu_D$ it will always remain so, and the spectral index remains adiabatic without change (until a much higher cutoff is reached where the acceleration becomes inefficient and the spectrum drops off exponentially). However for a homogeneous medium or one with $d < 4/3$, if initially $\nu_b > \nu_D$ the photon spectral index will at some later time steepen by 1/2 as ν_b sweeps through the observing band ν_D and the observed spectrum transitions from the adiabatic to the radiative regime.

4. Anisotropic Inhomogeneous Models

The observed afterglow temporal decays are conventionally fitted by power-laws, and it is interesting to explore how the decay slopes would depend on the angular dependence of the dynamically relevant quantities of a fireball. To that effect, we consider anisotropic relativistic outflows where both the energy per unit solid angle and the bulk Lorentz factor depend on the angle θ as power-laws (at least over some range of angles), and also consider the external density distribution to depend on radius as a power-law,

$$E \propto \theta^{-j} \quad , \quad \Gamma \propto \theta^{-k} \quad , \quad n \propto r^{-d} \; . \tag{10}$$

If there is a well defined jet, the normalizations of E and Γ may be different for material inside and outside the jet opening angle θ_o. At each angle the outflow starts converting a significant fraction of its bulk kinetic energy into radiation when an external blast wave develops at the angle-dependent deceleration radius $r_d \propto [E/n(r_d)\Gamma^2]^{1/3}$, at an angle-dependent observer-frame (detector) time $t \sim r/c\Gamma^2$. The θ-dependence of eq.(10) implies that the deceleration blast wave at different angles occurs at

$$\begin{aligned} \Gamma &\propto t^{-k(3-d)/(8k-j-2dk)} \; , \quad r \propto t^{(2k-j)/(8k-j-2dk)} \\ E &\propto t^{-j(3-d)/(8k-j-2dk)} \; , \quad \theta \propto t^{(3-d)/(8k-j-2dk)} \; . \end{aligned} \tag{11}$$

Depending on the normalization of (10) and causality considerations, the radiation from the blast waves occurring at increasing θ at successive times t can dominate the afterglow evolution (as opposed to the decay of E and Γ along the same θ as a function of time). For instance, if the event

has been detected at γ-rays, and there is a jet of opening angle θ_o, the observer is presumably within angles $\lesssim \theta_o$ from the axis. For $\Gamma = \Gamma_o(\theta/\theta_o)^{-k}$, in order for subsequent blast waves at $r = r_d$ from $\theta > \theta_o$ to be observed at times $t > t_o$ one needs $\Gamma^{-1} \gtrsim \theta$ to be satisfied, that is $\theta/\theta_o > (\theta_o\Gamma_o)^{1/(k-1)}$, or

$$t/t_o \gtrsim (\theta_o\Gamma_o)^{(8k-j-2dk)/[(3-d)(k-1)]} \; . \tag{12}$$

For values of $k < 1$ the blast waves are detectable at all angles, but for $k > 1$ there are ranges of k, j for which $\theta_o\Gamma_o$ is limited to values $\lesssim 5 - 10$ in order to detect the blast wave at reasonable t/t_o. In this case the initial part of the afterglow may be due to the evolution in time of the gas responsible for the burst initially observed, until such a time when the causality condition is satisfied for gas at larger angles, and the newly shocked gas at increasing angles can become dominant in providing the observed flux. This second case introduces additional complexities and will not be discussed here, since even the simpler case first mentioned above will serve to illustrate the point that a great variety can be expected in the temporal behavior of afterglows.

For the conditions where the afterglow is dominated by the newly shocked gas at increasing angles, the observer-frame peak frequency of the synchrotron radiation spectrum from the blast waves (11) coming from increasingly larger θ at increasing times t is

$$\nu_m \propto \xi_B\xi_e^2 \; n^{1/2}\Gamma^4 \;\; \propto \;\; t^{-[12k-d(3k+j/2)]/[8k-j-2dk]} \; . \tag{13}$$

The observer-frame intensity at this peak frequency is $I_{\nu_m} = \Gamma^3 I'_{\nu'_m} \sim \Gamma^2(E e_{sy,m}/4\pi r^2 t\nu_m) \propto \xi_B^{-1}\xi_e^{-2} e_{sy,m} \; t^{-d(k-j/2)/(8k-j-2dk)}$, where as in §3 the synchrotron efficiency is $e_{sy,m} \sim (t'_{ex}/t'_{sy,m})^a \propto (\xi_B^2\xi_e r^{1-d}\Gamma^2)^a$ if synchrotron and adiabatic cooling are the two most important energy loss mechanisms (or its generalization if IC or other effects need to be included). We have then $e_{sy,m} \propto t^{-a(4k+j(1-d))/(8k-j-2dk)}$, where $a = 1(0)$ if the peak electrons in the deceleration blast wave at the angle corresponding to detector time t are adiabatic (radiative). (In this model the dominant radiation is produced at the initial deceleration blast wave for that θ, so a does not enter in the dynamics, only in the radiative efficiency of the initial blast wave). The flux observed at ν_m from the deceleration blasts at increasing θ is $F_{\nu_m} \sim t^2\Gamma^2 I_{\nu_m}$, or

$$\begin{aligned} F_{\nu_m} \;\; &\propto \;\; \nu_m^{-[4k-2j-d(k-j/2)-a(4k+j\{1-d\})]/[12k-d(3k+j/2)]} \\ &\propto \;\; t^{[4k-2j-d(k-j/2)-a(4k+j\{1-d\})]/[8k-j-2dk]} \; , \end{aligned} \tag{14}$$

which scales with $\xi_B^{2a-1}\xi_e^{a-1}$. Depending on the normalization of eqs (10), the flux (14) from increasing θ values can dominate the flux given by equations (4) or (7). (In other cases, one can approximate the evolution as the superposition of isotropic blast waves from individual θ, which could in some cases be dominated by that of the central jet region). At a fixed detector frequency ν_D, the observed flux corresponding to eq. (14) is

$$F_D \sim F_{\nu_m}(\nu_D/\nu_m)^\alpha \;\; \propto \;\; t^{[k(4+12\alpha)-2j-d\{k(1+3\alpha)-(j/2)(1-\alpha)\}-a\{4k+j(1-d)\}]/[8k-j-2dk]} \; , \tag{15}$$

which scales with $\xi_B^{2a-1-\alpha}\xi_e^{a-1-2\alpha}$. For characteristic spectra with α positive (negative) below (above) the break, this leads to detected fluxes which initially rise in time, and then decay.

However, a variety of behaviors are possible, including some where the flux after the break passes through the detector window continues to grow at a slower rate, or saturates. Note that the scaling of eq. (14) with ν_m allows both for F_{ν_m} to decrease or to increase as ν_m decreases in time, both in the radiative and adiabatic cases, depending on the values of j and k which characterize the angular dependence of E and Γ.

5. Discussion

5.1 Dynamics, Cooling and Decay Law.- In the simplest model where the GRB and the afterglow both arise from an external shock (case a1 of Mészáros & Rees, 1997a), to zeroth order the GRB γ-ray flux should lie near the backward extrapolation of the afterglow, provided the basic conditions have not changed and the same radiation mechanisms are responsible for both. This is clearly a rough approximation, since it is likely that the GRB is initially radiatively efficient, and becomes radiatively inefficient at some later stage. The observations of GRB 970228, especially following the HST observations of September 1997 (Fruchter, et al, 1997) indicate an optical flux decaying as an approximately constant power law in time. Superposed on this overall long-term behavior, there may be shorter timescale variations (e.g. Galama, et al, 1997), which could be either a difference in calibration, or a real wiggle in the decay.

At late times (e.g. six months in GRB 970228) the remnant is most likely adiabatic. One question that can arise is whether a simple external shock afterglow model whose dynamics is manifestly "adiabatic" can be radiatively efficient enough to produce the initial relatively high X-ray and optical afterglow luminosity. For an afterglow luminosity $L_{X,O} \lesssim L_\gamma$, as observed, from our discussion in §2 this is not a problem. At each radius, the electrons can radiate up to half of the total newly shocked proton energy randomized in the shock transition, as long as the electron cooling time is shorter than the expansion time. The electrons can be radiatively efficient even when the dynamics of the remnant (i.e. the shell of hot protons and magnetic fields behind the shock, which provide most of the mass and inertia) follows an "adiabatic" law $\Gamma \propto r^{-3/2}$ for a homogeneous medium. Arguments were presented §2 why the latter behavior may be more likely than a faster evolution with $\Gamma \propto r^{-3}$, a conclusion also supported by comparison with observations relating to the size of the GRB 970228 remnant (Waxman, et al, 1997c). Note also that, from equation (2), for expansion in medium whose density decreases, Γ could drop even more shallowly than the above for a homogeneous medium. As shown by Tavani, 1997;Waxman, 1997a; Wijers, Rees & Mészáros , 1997; Reichart, 1997 and others, in GRB 970228 the initial γ-ray flux and the overall X/O afterglow behavior are in good agreement with a simple external shock where the observations started after $\nu_m < \nu_D$, without any substantial changes of slope during the observed decay phase. However, as discussed below, there are various mechanisms capable of producing changes in the decay slope, which could be responsible for some of the reported departures from a simple power law behavior.

5.2 Intensity Offsets Between Afterglows and Main Burst .- In the case of GRB 970228, a

backward extrapolation of the afterglow flux shows some hints of undershooting the γ-ray flux. If this were real, a slight undershooting could be due to the GRB being radiative initially (this is expected especially during the initial deceleration shock), with the afterglow dynamics becoming adiabatic soon afterwards. This would lead to a flattening of the spectral slope (e.g. second line, last two columns of Table 2).

Another possibility (Mészáros & Rees, 1997a; Wijers, Rees & Mészáros , 1997; Katz & Piran, 1997) is that the γ-rays could have originated in an internal shock. Internal shocks leave essentially no afterglow, yet they should be followed (eventually) by external shocks. An internal shock leaves unused anywhere from $\sim 20\%$ to $\gtrsim 90\%$ of the total kinetic energy (Rees & Mészáros , 1994; Kobayashi, Sari & Piran, 1997). The leftover energy is liberated in the external shock, most of whose initial radiation can come out at GeV energies because initially inverse Compton (IC) losses dominate over synchrotron losses; the initial synchrotron MeV radiation would then be typically low below the BATSE threshold (Mészáros & Rees, 1994), but later on it becomes dominant over IC (Waxman, 1997b). Thus at later stages the afterglow from the synchrotron peak can have a flux level whose back extrapolation might overshoot the γ-ray flux. Since external shocks are generally smoother (at most 3-5 pulses, Panaitescu & Mészáros , 1997), while internal shocks may be very variable (Rees & Mészáros , 1994;Kobayashi, Sari & Piran, 1997), a natural conclusion (also reached independently by Piran, 1997) is that since afterglows appear to arise from external shocks, a burst where the gamma-ray light curve (or at least the last gamma-ray pulse of the light curve) is relatively smooth has a better chance of leaving behind a visible afterglow at lower frequencies.

A reason for offsets may also be (§4) that the relativistic outflow has an angle dependence in either the energy, the bulk Lorentz factor, or both. The GRB itself may be due, for instance, to a high Γ ejecta which shocks first, while the afterglow could be dominated by slower material ejected at larger angles relative to the observer, which shock later and produce softer radiation, but which could carry a substantial or even larger fraction of the total energy. If detected this would generally be as an upward offset of the afterglow relative to the GRB, since otherwise the afterglow would be dominated by the evolution of the same material which gave rise to the GRB.

5.3 Rise and Decay, Late Rises, Bumps .- An initial afterglow flux rise followed by a decay is a direct consequence of the simplest afterglow model (a1), and is true also of any generic peaked spectrum from an expanding cloud where the peak energy decreases in time faster than the peak flux. Estimates for expanding clouds were made by Paczyński & Rhoads, 1993 and Katz, 1994b based on simplified radiation models. In the more detailed model (a1) of Mészáros & Rees, 1997a, if the slope α at frequencies below ν_m is positive, $F_\nu \propto \nu^\alpha$, the initial rise is $F_D \propto t^{3\alpha/2}$ for $\nu_D < \nu_m$ in a homogeneous medium (Wijers, Rees & Mészáros , 1997). For an "average" GRB spectrum, $\alpha \sim 0$ below the break (Band, et.al., 1993), which implies F_{ν_D} initially constant (Mészáros & Rees, 1997a). However, $\alpha \sim 0$ is only the average value; there are many GRB with $\alpha > 0$ below the break (this is also the case for an ideal synchrotron spectrum $\alpha \sim 1/3$ below the break, e.g. Mészáros , Rees & Papathanassiou, 1994), and in such cases one obtains an initial

power law increase in F_D. After the frequency ν_m of the peak has dropped below the observing frequency ν_D, if the spectrum above the peak is $F_\nu \propto \nu^\beta$ with $\beta < 0$, the flux observed at ν_D starts to decay $\propto t^{-3\beta/2}$. This occurs after a time $t_D/t_\gamma \sim (\nu_\gamma/\nu_D)^{2/3}$ (for an adiabatic remnant $\Gamma \propto t^{-3/8}$), which for observations in the R-band is $t_{opt} \sim 10^4 t_\gamma$. A maximum at 1.5 days such as in GRB 970508 is therefore compatible with the observed $t_\gamma \sim 5 \times 10^1$ s.

However, there are other mechanisms which can give rise to optical fluxes rising at times which could be even later than the above. One is an anisotropic outflow (§4), where the GRB and X-rays come from material close to the axis oriented near the l.o.s., and further off-axis material with a slower Γ starts to decelerate after 1.5 days, say, or it has been slowed down enough that its light cone includes the l.o.s. after 1.5 days. Since beaming does not change the slope, if the spectrum is a pure power law then its slope would remain constant. While a constant spectral slope is the simplest assumption, it is not a necessary one. It is conceivable, for instance, that the slope of accelerated electrons (and consequently of the synchrotron spectrum) could change as the bulk Lorentz factor and the shock strength changes, or it might depend on other effects associated with the angular anisotropy of the outflow. An interesting result is the indication that the optical light curve of GRB 970508 may have been steady or even decreasing (Pedersen, 1997) before the 1.5 day rise phase preceding the maximum. This may be simply explained in an anisotropic model such as described in §4 where the indices j and k change to give a light curve transition in this sense, or even more simply, by a bimodal model where one has a central jet associated with the γ-ray event, whose tail is just seen to decay, followed by the emission of a slower Γ outflow over much wider angles outside the jet which is responsible for the main part of the afterglow. Another possibility for a late rise in the optical would be if a lower Γ shell catches up with the main shock front with a comparable energy after $t \sim 1.5$ days, so we see then the emission from the onset of deceleration of this late shell. Very prolonged optical decays with a shallow power law are possible in anisotropic models such as in §4, e.g. Table 2, lines 6 or 7.

An alternative explanation for a late turn-on of an afterglow may be that the GRB occurs inside a very low density cavity inflated by the pulsar activity of one of the neutron stars in the progenitor binary. The shock, at least over a range of directions, would not arise until the ejecta hits the wall of the cavity, and this could take a time of order weeks, the ensuing shock being spread over a dynamic time scale sufficiently short to produce a large flux per unit time. A characteristic feature of pulsar cavities is that they are usually asymmetric and irregular in shape, often being elongated due to the proper motion of the energizing source. One would naturally expect a wide variety of time histories for the afterglows arising from the impact of a (possibly anisotropic) ejecta upon an irregularly shaped cavity wall whose dimension (depending on direction respect to the line of sight) may vary considerably.

5.4 Peak Flux Level Evolution.- In some observed afterglows the flux level F_ν at lower energies is, at least initially, significant relative to that of the maximum gamma-ray flux (even if νF_ν is smaller). One prediction of the simplest model is that the maximum value of the afterglow flux in every band, F_{ν_m}, is a constant. An interesting case is that of GRB 970508, where the ratio

of maximum F_X to maximum F_γ is $\sim 1/2$, while the ratio of F_O to F_γ is $\sim 1/10$. Considering the drastic simplifications involved in the simplest model (homogeneous medium, constant equipartition fields, etc), this order of magnitude agreement is perhaps encouraging. However, $F_{\nu_m} \sim$ constant is clearly an approximation which need not always hold for less simple models. In Table 1, the top four lines of the last two columns show that for isotropic but inhomogeneous outflows one can expect F_{ν_m} to either increase or decrease with ν_m as the later decreases, while the previous columns of the top four lines in the same Table show that either of these behaviors may also arise as a result of an anisotropic outflow. Thus, the ratio of F_{ν_m} at γ-rays and lower wavelengths could be either larger or smaller than one, simply on this basis. In a simple bimodal anisotropic model, one can simply have more or less energy in the large angle slower outflow seen later than in the early and harder axial outflow.

A related question is the observability of radio fluxes of order mJy around 10^{10} Hz, as reported for GRB 970508 (Frail, et al, 1997). Radio fluxes of this magnitude arise naturally in a simple isotropic homogeneous fireball model after a week or so, since the self-absorption frequencies (overestimated by 10^2 in Mészáros & Rees, 1997a) are in the range of 10^{11} Hz initially and drop to 10^{10} Hz in about a week, and the flux level is well below the brightness temperature limits for incoherent synchrotron radiation. What is more interesting is the relatively large value of the radio flux ($\sim$ mJy) relative to the O flux ($\sim 50\mu$Jy). In this afterglow therefore F_{ν_m} first decreases with ν_m between γ and O energies, but then increases with decreasing ν_m between O and the R(adio) band. The simplest explanation may be in terms of a jet-wind two component model. The decrease between γ and O could be due to expansion of a jet into an inhomogeneous medium (e.g. Table 1, fourth line), and the increase between O and R could be due to a surrounding low Γ wind at larger angles, which shocks at later times, as suggested in Wijers, Rees & Mészáros , 1997. The slow wind need not have an angular dependence; a growth and decay of the flux is approximated also by behaviors represented in Table 2 in the last two columns for expansion into either a homogeneous or an inhomogeneous external medium. A wind with $\Gamma \sim 3-10$ can match the delayed emergence ($\sim$ week) of the radio from the wind blast wave. The tables give only illustrative values for selected spectral and density exponents, which can be easily changed to fit a particular observed rate of growth and decay. Actually, the radio flux of GRB 970508 at 10^{10} Hz (Frail et al 1997) at first increased and then appears to flatten, except for decaying oscillations which could be due to scintillation (Goodman, J., 1997,Waxman, et al, 1997c), followed by a slow decline.

5.5 Burstless Afterglows and Afterglowless Bursts.- An interesting consequence of anisotropic models (Mészáros & Rees, 1997b,Rhoads, 1997) is that there could be a large fraction of detectable afterglows for which no γ-ray event is detected. If the observer lies off-axis to the jet, then the detected "afterglow" can be approximated by the isotropic model calculated for an $E(\theta_{obs})$, $\Gamma(\theta_{obs})$ corresponding to the offset angle θ_{obs} of the observer to the jet axis. As Γ drops after the deceleration shock, the causal angle includes an increasing amount of the solid angle towards the jet as well as towards the equator, and depending on the values of j and k the observed flux would

generally decrease in time. However for some choices of j, k an increase in the light curve might be possible, depending on the normalization. It could be that gravitational energy is converted more efficiently into kinetic energy of expansion at large angles, where the opacity is larger (c.f. Paczyński, 1997). After $\Gamma(\theta_{obs})$ has dropped to the point where the central jet portion $\theta \lesssim \theta_o$ is detectable, the late stages of the jet emission would become visible, at a later stage when it is bright only at wavelengths longer than γ-rays. If the jet contained substantially more energy than the off-axis regions so that it dominates the flux even after expanding for a longer time than the initially observed off-axis region, one would expect an additional increase or flattening of the light curve at this point. Details would be further complicated by contributions from the equator and the back side of an opposite jet, if $\theta_{obs} \gtrsim \pi/4$. The statistics of afterglows not detected in γ-rays can be calculated from equations (11, 14).

The converse question is why some bursts (e.g. GRB 970111) have been detected in γ-rays but not in X or O, even though it was in the field of view of Beppo-SAX, which would have been expected to detect it if the X to γ-ray ratio had been comparable to GRB 970228 (a weak X-ray afterglow may have been detected, Costa 1997b). One reason may be if the γ-ray emission is due to internal shocks (which leave essentially no afterglows, Mészáros & Rees, 1997a), and the environment has a very low density, in which case the external shock can occur at much larger radii and over a much longer time scale than in usual afterglows and the X-ray intensity is below threshold for triggering. This may be the case for GRB arising from compact binaries which are ejected from the host galaxy into an an external environment which is much less dense that the ISM assumed for usual models. Another possibility for an unusually low density environment, made up only of very high energy but extremely low density electrons, is if the GRB goes off inside a pulsar cavity inflated by one of the neutron stars in the precursor binary. Such cavities can be as large as fractions of a parsec or more, giving rise to a deceleration shock months after the GRB with a consequently much lower brightness that could avoid triggering and detection.

The lack of an afterglow in some bursts may also be due to occurrence in an unusually high density environment (e,g, a star-forming region, or the inner kiloparsecs of a late type spiral, where failed supernova or hypernova progenitors may reside, e.g. Paczyński, 1997). This could lead to a more rapid onset of the deceleration leading to the X-ray phase, and it would also imply an increased neutral gas column density and optical depth in front of the source. A special case is that of GRB 970828, where X rays have been observed, but no optical radiation down to faint levels (Groot et al. 1997). The presence of a significant column density of absorbing material has been inferred from the low energy turnover of the X-ray spectrum (Murakami et al., 1997), and the corresponding dust absorption may in fact be sufficient to cause the absence of optical emission (Wijers & Paczyński, private communications). The difference between the low density and high density environments cases could be tested if future observations of afterglows reveal a correlation with the degree of galaxy clustering or with individual galaxies.

In conclusion, the absence of detected afterglows in many bursts is not surprising, while there may be detected afterglows also in some cases where a corresponding gamma-ray burst has not

been detected; and when afterglows are detected, a wide diversity of behaviors may be the rule, rather than the exception.

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a	d	$F_{\nu_m},\ \nu_m$	$j=$ $3k$	$j=$ $2k$	$j=$ k	$j=0$ $k=1$	$j=$ $-k$	$j=$ $-2k$	$j=$ $-3k$	Isotr A=a	Isotr A=1
0	0	$F_{\nu_m} \propto \nu_m^q$	1/6	0	-1/6	-1/3	-1/2	-2/3	-5/6	-1/6	-1/3
0	2	$F_{\nu_m} \propto \nu_m^q$	1/3	0	-1/5	-1/3	-3/7	-1/2	-5/9	-1/5	-1/3
1	0	$F_{\nu_m} \propto \nu_m^q$	3/4	1/2	1/4	0	-1/4	-1/2	-3/4	0	0
1	2	$F_{\nu_m} \propto \nu_m^q$	2/3	1/2	2/5	1/3	2/7	1/4	2/9	1/3	1/3
0	0	$\nu_m \propto t^p$	-12/5	-2	-12/7	-3/2	-4/3	-6/5	-12/11	-12/7	-3/2
0	2	$\nu_m \propto t^p$	-3	-2	-5/3	-3/2	-7/5	-4/3	-9/7	-5/3	-3/2
1	0	$\nu_m \propto t^p$	-12/5	-2	-12/7	-3/2	-4/3	-6/5	-12/11	-3/2	-3/2
1	2	$\nu_m \propto t^p$	-3	-2	-5/3	-3/2	-7/5	-4/3	-9/7	-3/2	-3/2

Table 1: Exponents of the power law dependence of the synchrotron peak flux F_{ν_m} as a function of the time-varying synchrotron peak ν_m (top), and exponents of the time dependence of ν_m on observer (detector) time t (bottom). The first column indicates whether the electrons are in the radiative ($a = 0$) or adiabatic ($a = 1$) regime, and the second column indicates the value of the exponent of the external medium density dependence on radius $n \propto r^{-d}$, $d = 0$ being homogeneous. Columns 4 through 11 give the exponents for the anisotropic model $E \propto \theta^{-j}$, $\Gamma \propto \theta^{-k}$ of §4 and various values of j and k. The last two columns on the right gives the corresponding exponents for the isotropic models of §3, the first being for the strong electron-proton coupling $a = A$ case, and the second for the weak coupling case $A = 1$. For $A = 0(1)$ the remnant as a whole is dynamically radiative (adiabatic).

			$F_D \propto t^w$								
a	d	α	$j=$ $3k$	$j=$ $2k$	$j=$ k	$j=0$ $k=1$	$j=$ $-k$	$j=$ $-2k$	$j=$ $-3k$	Isotr A=a	Isotr A=1
0	0	1/3	2/5	2/3	6/7	1	10/9	6/5	14/11	6/7	1
0	0	-1	-14/5	-2	-10/7	-1	-2/3	-2/5	-2/11	-10/7	-1
0	2	1/3	0	2/3	8/9	1	16/15	10/9	8/7	8/9	1
0	2	-1	-4	-2	-4/3	-1	-4/5	-2/3	-4/7	-4/3	-1
1	0	1/3	-1	-1/3	1/7	1/2	7/9	1	13/11	1/2	1/2
1	0	-1	-21/5	-3	-15/7	-3/2	-1	-3/5	-3/11	-3/2	-3/2
1	2	1/3	-1	-1/3	-1/9	0	1/15	1/9	1/7	0	0
1	2	-1	-5	-3	-7/3	-2	-9/5	-5/3	-11/7	-2	-2

Table 2: Exponents of the power law dependence of the synchrotron flux in a given detector band ν_D as a function of observer (detector) time t. The first column indicates whether the electrons are radiative ($a = 0$) or adiabatic ($a = 1$), the second column gives the exponent of the external medium density dependence on radius $n \propto r^{-d}$, $d = 0$ being homogeneous, column 3 gives the value of the spectral index α of the spectrum, $F_\nu \propto \nu^\alpha$, which may be 1/3 below the peak ν_m, and -1 above (although these indices can vary around these representative values). As the peak ν_m passes down through the observation band ν_D the index α changes from the positive value left of the peak to the negative to the right of the peak. Columns 5 through 12 give the time exponents of the $F_D \propto t^w$ in the detector band for the anisotropic model $E \propto \theta^{-j}$, $\Gamma \propto \theta^{-k}$ of §4 and various values of j and k. The last two columns on the right give the corresponding exponents for the isotropic models of §3, the first being for the strong coupling case $a = A$ and the second for the weak coupling $A = 1$. For $A = 0(1)$ the remnant as a whole is dynamically radiative (adiabatic).